銘傳大學 100 學年度研究所碩士班招生考試

電腦與通訊工程學系碩士班

第二節

線性代數試題

（第 1 頁共 1 頁）（限用答案本作答）　　☐可使用計算機　☑不可使用計算機

1. Find all values of a for which the resulting linear system has (a) no solution, (b) a unique solution, and (c) infinitely solution. (12%)

$$\begin{aligned} x+y-z&=2\\ x+2y+z&=3\\ x+y+(a^2-5)&=a \end{aligned}$$

2. Given $A=\begin{bmatrix}1&2&3\\1&1&2\\0&1&2\end{bmatrix}$, find the inverse of the matrix A. (8%)

3. Let $A=\begin{bmatrix}2&2&3\\1&2&1\\2&2&1\end{bmatrix}$, find the eigenvalues and eigenvectors.
   Find the matrix P such that $P^{-1}AP=D$, where D is a diagonal matrix.
   Then find A^5. (32%)

4. Compute the basis for the row space of A and rank of A. (12%)

$$A=\begin{bmatrix}1&-2&5\\2&3&2\\0&-7&8\end{bmatrix}$$

5. Let $L: R^3 \to R^3$ be a linear transformation such that $L(i)=\begin{bmatrix}1\\2\\-1\end{bmatrix}$, $L(j)=\begin{bmatrix}1\\0\\-2\end{bmatrix}$, $L(k)=\begin{bmatrix}1\\1\\3\end{bmatrix}$, where $i=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $j=\begin{bmatrix}0\\1\\0\end{bmatrix}$, $k=\begin{bmatrix}0\\0\\1\end{bmatrix}$. Find $L\left(\begin{bmatrix}2\\-1\\3\end{bmatrix}\right)$. (10%)

6. Solve the linear system and write the solution x as $x=x_p+x_h$, where x_p is a particular solution and x_h is a homogeneous solution. (14%)

$$\begin{aligned} x+2y-z-w&=3\\ x+y+3z+2w&=-2\\ 2x-y+4z+3w&=1\\ 2x-2y+8z+6w&=-4 \end{aligned}$$

7. Do the polynomials $t^3+2t+1,\ t^2-t+2,\ t^3+2,\ -t^3+t^2-5t+2$ span P_3? (Hint: Check whether these vectors are linearly independent or not) (12%)

試題完