銘傳大學八十八學年度資訊管理研究所碩士班招生考試
第二節

離散數學 試題

每大題 10 分。請把握時間，簡明扼要作答。

1. (a) Let p, q, r denote primitive statements. Please simplify the following statement:
   ~((~p∨q)^r)->~q
   (b) Fig.1 illustrates the diagram of a switching network which can be represented as a compound statement:
   (p∨q∨r)^(p∨~t∨r)^(p∨t∨~q)

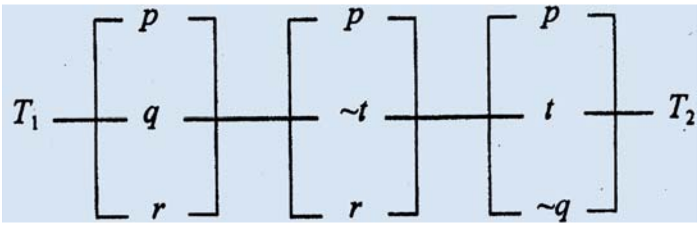

Fig.1 A switching network
Please depict the diagram of the simplified switching network.

2. Let p(x), q(x), and r(x) demote the following open statements.

p(x): $x^2 - 8x + 15 = 0$
q(x): x is odd
r(x): x > 0

For the universe of all integers, determine the truth or falsity of each statement. If a statement is false, give a counterexample.

(a) ∀x[p(x)->q(x)]
(b) ∀x[q(x)->p(x)]
(c) ∃x[p(x->q(x)]
(d) ∃x[q(x)->p(x)]
(e) ∃x[r(x)->p(x)]
(f) ∀x[p(x)->r(x)]
(g) ∃x[r(x)->p(x)]
(h) ∀x[~q(x)->~p(x)]
(i) ∃x[p(x)->(q(x)^r(x))]
(j) ∀x[p(x)∨q(x))->r(x)]

3.
（a） Solve the recurrence relation defined by: $a_0 = a_1 = 3$, $a_n = a_{n-1} + 2a_{n-2}$, for all $n \in N$.
（b） Let T_n denote the number of movements of discs in the Hanoii Tower problem with *n* discs. Define the recurrence relation for the recursive algorithm that you may design. Solve the recurrence relation

for T_n.

4.

(a) What is a partial order?

(b) Let *R* be a partial order on a set A and R^{-1} be the inverse of R. Show that R^{-1} is a partial order.

5. Prove that a function f: A->B is invertible *if and only if* it is one-to-one and onto.

6. A weighted graph is shown in Fig.2.

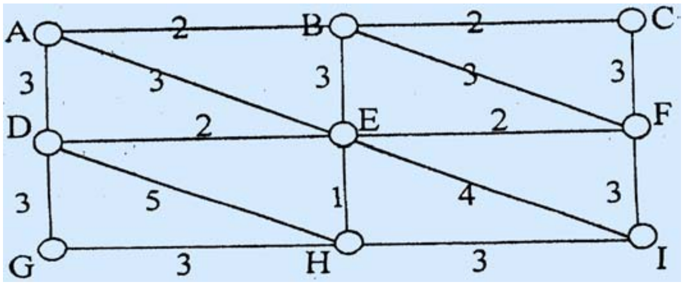

Fig. 2 A weighted graph

(a) Find a minimum spanning tree and its weight.

(b) Describe the algorithm you used.

(c) What is the time complexity of your algorithm？Explain.

(d) Give a real-world example for possible application of minimum spanning tree.

7. Which of the following three relations is(are) an equivalent relation(s)? Explain.

(a) Let S be a set of cars. For x, y $\in$ S, x αy if x and y are of the same color;

(b) For x, y $\in$ N, xηy if x – y is odd;

(c) For x, y $\in$ N$\cup${0}, xψy if (x mod 5) and (y mode 5) have the same remainder.

8.

(a) What is Pigeon-Hole Principle?

(b) Suppose 98 balls are placed in 6 boxes. Show that there must be some box containing at least 17 balls.

(c) Let A be a subset of {1, 2, 3,…,149, 150} and |A| = 25. Show that there are two disjoint pairs of numbers of A having the same sum (for example, {3, 89}) and {41, 51} have the same sum 92). Hint: Count the total number of two-element subsets of set A.

9. Consider the poset (S, R) where S ≡ {1,2,3,4,5,6,10,12,15,20,30,60} and R is the relation of divisibility
   (a) Please draw the Hasse diagram of (S, R).
   (b) Show that (S, R) is a lattice.

10. Explain the following terms:
    (a) compatibility relation
    (b) strongly connected
    (c) hamiltonian circuit
    (d) minimal dominating set

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