銘傳大學八十九學年度資訊管理研究所碩士班招生考試
第二節

離散數學 試題

第一至第八大題，每題 10 分；第九大題 20 分

1. (a) Let p, q, r denote primitive statements. Please simplify the following statement:

$$(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)$$

(b) Fig.1 illustrates the diagram of a switching network which can be represented as a compound statement:

$(p \vee q \vee r) \wedge (p \vee \sim t \vee r) \wedge (p \vee t \vee \sim q)$

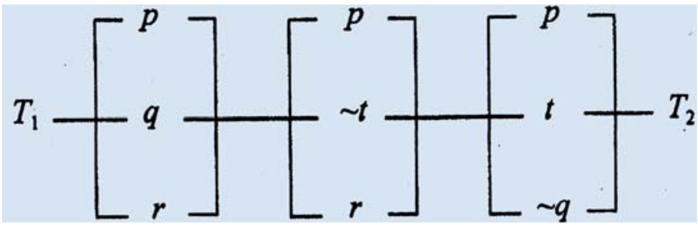

Fig.1 A switching network

Please depict the diagram of the simplified switching network.

2. For the following statements the universe comprises all nonzero integers. Determine the truth or falsity of each statement. If a statement is false, give a counterexample.

(a) $\exists x \exists y[xy = 1]$

(b) $\exists x \forall y[xy = 1]$

(c) $\forall x \forall y[\sin^2 x + \cos^2 x = \sin^2 y + \cos^2 y]$

(d) $\exists x \exists y[(3x - y = 7) \wedge (2x + 4y = 3)]$

Let the universe for the variables in the following statements consist of all real numbers. Please negate and simplify these statements.

(e) $\forall x \forall y[(x < y) \rightarrow \exists z(x < z < y)]$

(f) $\lim_{x \to a} f(x) = L \Leftrightarrow \forall \varepsilon > 0 \ \exists \delta > 0 \ \forall x((0 < |x - a| < \delta) \rightarrow (|f(x) - L| < \varepsilon)\ ]$

3. (a) The sequence of the Lucas numbers is defined recursively by

(1) $L_0=2$, $L_1=1$; and

(2) $L_n=L_{n-1} + L_{n-2}$, for $n \varepsilon Z^+$ with $n \geqq 2$

Prove that for n$\geqq$N

$$\sum_{i=0}^{n} L_i = L_{n+2} - 1$$

(b) Let T_n denote the number of movements of discs in the Hanoi Tower problem with n discs. Define the recurrence relation for the recursive algorithm that you may design. Solve the recurrence relation for T_n.

4. For each of the following statements about relations on a set A, where |A| = n, determine whether the statements are true of false. If it is false, give a counterexample.
   (a) If R is a reflexive relation on A, then |R|$\geqq$n.
   (b) If R is a relation on A and |R|$\geqq$n, then R is reflexive.
   (c) If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 reflexive=> R_2 reflexive.
   (d) If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 antisymmtric => R_2 antisymmtric.
   (e) If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 reflexive => R_1 reflexive.
   (f) If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 transitive => R_1 transitive.

5. Let f: Z->N be defined by

$$f(x) = \begin{cases} 2x - 1, & \text{if } x > 0 \\ -2x, & \text{for } x \leqq 0 \end{cases}$$

   (a) Prove that f is one- to –one and onto.
   (b) Determine f^{-1}.

6. (a) Give the definition of a tree. (Suppose that G=(V, E) is a undirected graph.)
   (b) Prove that in any tree T=(V, E), |V| = |E| + 1.
   (c) Let T=(V, E) be a tree with |V| = n. How many distinct paths are there in T?
   (d) Describe an algorithm you used to find a minimum spanning tree. What is the time complexity of your algorithm? Explain.

7. Let a, b, c Z^+ with b$\geqq$2, and let f: Z^+ -> R. Show that if

$$f(1) = c, \text{ and}$$
$$f(n) = af(n/b) + c, \text{ for } n = b^k, k \geqq 1,$$

then for all n = 1, b, b^2, b^3, …,

(a) $f(n) = c(\log_b n + 1)$, when $a > 1$;

(b) $f(n) = c(an^{\log_b a}-1) / a - 1$, when $a \geqq 2$.

8. For the alphabet $\Sigma=\{0, 1\}$, let A, B, C Σ^* be the following languages:

A = {0,1,00,11,000,111,0000,1111}

B = $\{w \Sigma^*, \|w\| \geqq 2\}$, {$\|w\|$ denotes the length of a string w.}

C = $\{w\Sigma^*, 2 \geqq \|w\|\}$.

Find sentences below:

(a) A- B

(b) B∩C

(c) B∪C

(d) $\overline{(AIC)}$

9. Give definitions for the following terms:

(a) partial ordering

(b) Pigeon-Hole Principle

(c) Hamiltonian cycle

(d) graph isomorphism

(e) maximal independent set:

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