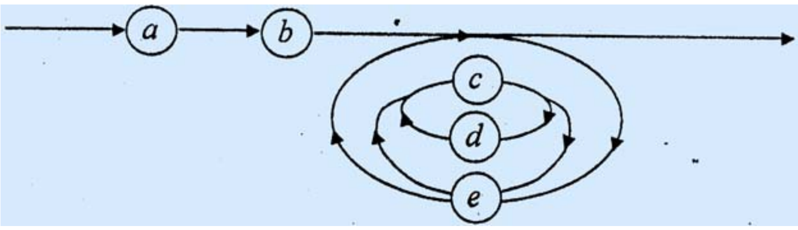
銘傳大學九十學年度資訊管理研究所碩士班招生考試
第二節

離散數學 試題

1. (20pts.) Give definitions for the following terms：
   (a) Tree
   (b) Group
   (c) Pigeon-Hole Principle
   (d) Hamiltonian cycle
   (e) Graph isomorphism

2. (15pts.)
   (a) Determine the language describe by the following syntax diagram. You may use the regular expression or BNF statements to represent your answer.



   (b) For the alphabet={0,1}，let A, B, C$\subseteq \Sigma^*$ be the following languages：
   A={ 0,1,00,11,000,111,0000,1111 },
   B=$\{w \in \Sigma^*, \|w\| \geqq 2\}$,($\|w\|$ demotes the length of a string w.)
   C=$\{w \in \Sigma^*, 2 \geqq \|w\|\}$
   Find sentences below：
   (a) A-B
   (b) $B \cap C$
   (c) $B \cup C$
   (d) $\overline{A \cap C}$

3. (15pts.)
   (a) Let p, q, r denote primitive statements. Please simplify the statement：
   $$\sim(((p \vee q) \wedge r) \rightarrow \sim q)$$
   (b) Find the negation of the statement below.
   $$\lim_{x \to a} f(x) = L \Leftrightarrow \forall \varepsilon > 0 \exists \delta > 0 \forall x((0 < |x - a| < \delta) \rightarrow (|f(x) - L| < \varepsilon))$$
   (c) Let n be an integer. Prove that if n^2 is odd, then n is odd.

4. (10pts.) Let f：Z→N be defined by

$$f(x)= \begin{cases} 2x-1, \text{ if } x>0 \\ -2x, \text{ for } x\leqq 0 \end{cases}$$

(a) Prove that f is one-to-one and onto.
(b) Determine f^{-1}

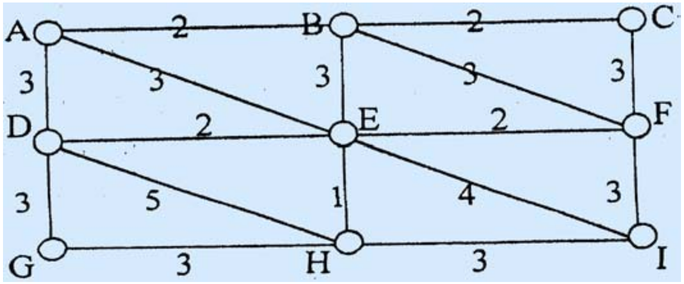
5. (10pts.) For each of the following statements about relations on a set **A**, where |**A**| = n, determine whether the statements is true or false. If it is false, give a counterexample.
   (a) If R is a reflexive relation on **A**, then $|R|\geqq n$.
   (b) If R is a relation on **A** and $|R|\geqq n$, then R is reflexive.
   (c) If R_1, R_2 are relations on **A** and $R_2\supseteq R_1$, then R_1 is reflexive=>R_2 reflexive.
   (d) If R_1, R_2 are relation on **A** and $R_2\supseteq R_1$, then R_1 antisymmtric=>R_2 antisymmtric.
   (e) If R_1, R_2 are relations on **A** and $R_2\supseteq R_1$, then R_2 reflexive=>R_1 reflexive.
   (f) If R_1, R_2 are relations on **A** and $R_2\supseteq R_1$, then R_2 transitive=>R_1 transitive.

6. (10pts.)
   (a) The sequence of the Fibonacci numbers is defined recursively by
      (1) $F_0=0$, $F_1=1$; and
      (2) $F_n=F_{n-1}+F_{n-2}$, for $n\in Z^+$ with $n\geqq 2$.
      Prove that for $n\in Z^+$

$$\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$$

   (b) Let T_n denote the number of movements of discs in the Hanoi Tower problem with n discs. Define the recurrence relation for the recursive algorithm that you may design. Solve the recurrence relation for T_n.

7. (10pts.) A weighted graph is shown below.



(a) Find a minimum spanning tree and its weight.
(b) Describe the algorithm you used.
(c) What is the time complexity of your algorithm？Explain.
(d) Give a real-world example for possible application of minimum spanning tree.

8. (10pts.) Consider the poset (S,R) where S≡{1,2,3,4,5,6,10,12,15,20,30,60} and R is the relation of divisibility.
(a) Please draw the Hasse diagram of (S,R).
(b) Show that (S,R) is a lattice.

試題完