銘傳大學九十一學年度資訊工程研究所碩士班招生考試

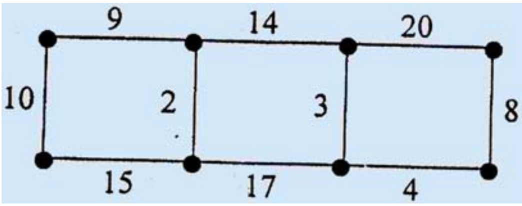
第四節

計算機數學 試題

1. In how many ways can we spell the word MATHEMATICS by a downward path starting at the top in the following array. (10%)

M
A A
T T T
H H H H
E E E E E
M M M M M M
A A A A A
T T T T
I I I
C C
S

2. Consider the weighted graph,



(a) Find a minimum-weight spanning tree of the graph. (5%)

(b) Explain the algorithm you used. (10%)

3. Find out the recursive definition function ($a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots + c_ra_{n-r}$) of the following sequence. (10%)

   1, 8, 27, 64, 125, 216, 343

4. The Fibonacci numbers Fn have the initial values $F_0 = 0$, $F_1 = 1$, and the recursion $F_n = F_{n-1} + F_{n-2}$ if $n \geqq 2$.

(a) Prove by induction that $\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}$ if $n \geq 1$. (5%)

(b) Prove by induction that $F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$ if $n \geq 0$.

(10%)

Hint: $\left(\frac{1+\sqrt{5}}{2}\right)^2 = \frac{1+\sqrt{5}}{2}+1$, $\left(\frac{1-\sqrt{5}}{2}\right)^2 = \frac{1-\sqrt{5}}{2}+1$

5. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$, find A^{-1}. (10%)

6. Prove the following statements.

   (a) If $A = [a_{ij}]$ is an n x n matrix and x and y are vectors in R^n, Then

   $$(Ax, y) = (x, A^T, y)$$

   (b) For any two vectors in an inner product space, we have

   $$||u + v||^2 + ||u - v||^2 = 2||u||^2 + 2||v||^2$$

7. Let W be the subspace of R^4 with the standard inner product with basis S = $\{u_1, u_2, u_3\}$, where

   $$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

   (a) Transform S to an orthonormal basis T = $\{w_1, w_2, w_3\}$. (10%)

   (b) Find the orthogonal projection of

   $$v = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 4 \end{bmatrix}$$

   on span T with respect to T. (10%)

8. Let L: $R^3 \rightarrow R^3$ be the linear transformation defined by $L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} a_1 + a_3 \\ a_1 + a_2 \\ a_2 - a_3 \end{bmatrix}$

   (a) Find a basis for ker L. (5%)

   (b) Find a basis for range L. (5%)

試題完