資訊傳播工程學系

銘傳大學九十二學年度　　　　　　碩士班招生考試

資訊工程學系

第四節

線性代數 試題

一、Let A=$\begin{bmatrix} \alpha & 0 & \beta & 2 \\ \alpha & \alpha & 4 & 4 \\ 0 & \alpha & 2 & \beta \end{bmatrix}$ **be the augmented matrix for a linear system. For what values of** α **and** β **does the system have**

(1) a unique solution,　(2)a one-parameter solution,

(3) a two-parameter solution, (4) no solution.　　**20%**

二、**Let A be 3×3 matrix whose eigenvalues are –3, 4, and 4, and associated eigenvectors are** $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$**, and** $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$**, respectively.**

(1) Find the matrix A.　　**10%**

(2) Compute A^9　　**10%**

三、**Let** w **be the space spanned by** $f=\sin$ **and** $g=\cos x$.

(1) Show that for any value of $\theta, f_1=\sin(x+\theta)$ **and** $g_1=\cos(x+\theta)$ **are vectors in** w.　　**5%**

(2) Show that f_1 **and** g_1 **form a basis for** w.　　**10%**

四、**Let A be n×n matrix and** $adjA$ **be the adjoint of** A.

(1) Show that $\det(adjA)=(aet(A))^{n-1}$.　　**7%**

(2) Find $adj(adjA)$.　　**8%**

五、**Let** $L: R^n \to R^n$ **be a linear operator defined by** $L(x)=Ax$ **, for** x **in** R^n**. Show that** L **is a one-to-one and onto if and only if** A **is nonsingular(invertible).**　　**10%**

五、**Let** W **be subspaces of** R^3 **with orthonormal basis** $\{w_1, w_2\}$**, where** $w_1=(0,1,0)$ **and** $w_2=(\frac{1}{\sqrt{5}},0,\frac{2}{\sqrt{5}})$.

(1) Write the vector $v_1=(1,2,-1)$ **as** $w+u$**, with** w **in** W **and** u **in**

$W^{\perp}$.$W^{\perp}$ **is the orthogonal complement of** W**.** **10%**

(2) Find the distance from $v_2 = (-1,0,1)$ **to** W**.** **10%**