

線性代數 試題

(限用答案本作答)

一、Consider the linear system

$$\begin{aligned} a^2x + 3z &= 1 \\ 5x + ay + 2z &= 0 \\ 3x + z &= 0 \end{aligned}$$

For what values of a does the linear system have a unique solution? Also, use Cramer's rule to find the solution. 15%

二、Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$ and $\det A = 3$, evaluate the determinant

(1) $\det(-2A)$ (2) $\det(A^{-1})$ (3) $\det(3A^{-1})^T$ (4) $\det(\text{adj}A)$

(5) $\det \begin{bmatrix} a+2p & -x & 3p \\ b+2q & -y & 3q \\ c+2r & -z & 3r \end{bmatrix}$ 15%

三、Prove or disprove that the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ has a LU-decomposition. 10%

四、Show that the set of all 3×3 symmetric matrices is a subspace W of all 3×3 matrices M_{33} . Find a basis for W . What is the dimension of W ? 20%

五、Suppose that $L: R^2 \rightarrow R^3$ is linear transformation and that

$L(1,2) = (8,4,3)$ and $L(3,4) = (18,6,7)$.

(1) Find the standard matrix representation A of L . 10%

(2) Is L one-to-one? 10%

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六、Orthogonally diagonalize the matrix $A = \begin{bmatrix} -1 & -4 & -8 \\ -4 & -7 & 4 \\ -8 & 4 & -1 \end{bmatrix}$; that is to
find matrices P and D such that $A = PDP^{-1}$. 20%

本試題係兩面印刷

試題完