離散數學 試題 （限用答案本作答）

(1) Ackermann's function, $A(m, n)$, is defined *recursively* for integers $m, n > 0$ by: (15%)

$$A(0, n) = n + 1, n > 0;$$
$$A(m, 0) = A(m-1, 1), m > 0;$$
$$A(m, n) = A(m-1, A(m, n-1)), m > 0, n > 0;$$

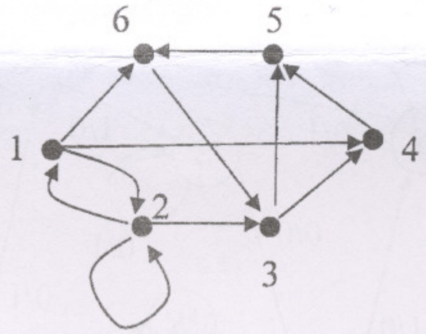
Calculate $A(2, 3)$.

(2) Use the directed graph in the following figure to answer these:

(a) Which nodes are reachable from 3?

(b) What is the length of the shortest path from 3 to 6?

(c) Give a path from 1 to 6 of length 8. (5% × 3 = 15%)



(3) For positive integers n and t, determine the coefficient of $x_1^{n_1} x_2^{n_2} x_3^{n_3} \ldots x_t^{n_t}$ in $(x_1 + x_2 + x_3 + \ldots + x_t)^n$, where each n_i is an integer with $0 \le n_i \le n$, for all $1 \le i \le t$, and $n_1 + n_2 + n_3 + \ldots + n_t = n$. (15%)

(4) (a) Let $G=(V, E)$ be a weighted connected undirected graph. If $e_1 \in E$ with *weight*(e_1) < *weight*(e) for all other edges $e \in E$, prove that edge e_1 is part of every minimal spanning tree for G.

(b) With G as in part (a), suppose that there are edges $e_1, e_2 \in E$ with *weight*(e_1) < *weight*(e_2) < *weight*(e) for all other edges $e \in E$, prove or disprove: Edge e_2 is part of every minimal spanning tree for G.

(10% × 2 = 20%)

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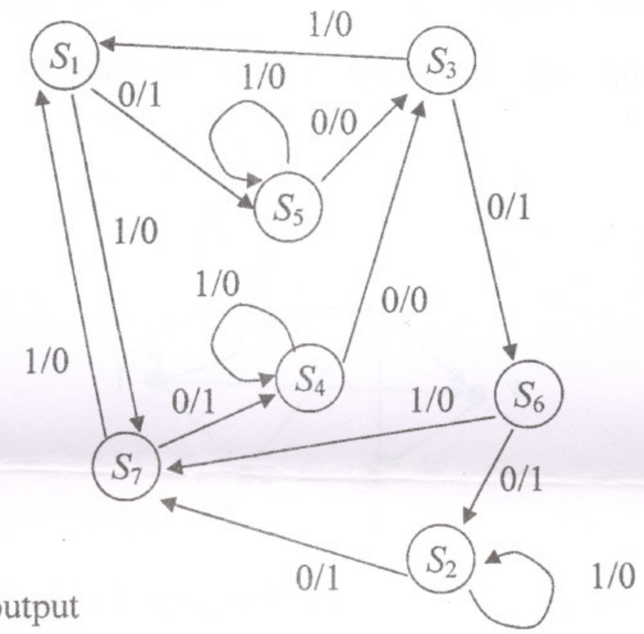
(5) Let M be the finite state machine given in the following state diagram. Apply the minimization process to find a minimal machine that is equivalent to it. (15%)



$x / y \equiv$ input / output

(6) Let $G=(V, E)$ be an undirected graph with adjacency matrix $A(G)$ as shown below. Use a breadth-first search based on $A(G)$ to determine if G is connected. (20%)

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	0	1	0	0	0	0	1	0
v_2	1	1	0	1	1	0	1	0
v_3	0	0	0	1	0	1	0	1
v_4	0	1	1	0	0	0	0	0
v_5	0	1	0	0	0	0	1	0
v_6	0	0	1	0	0	1	0	0
v_7	1	1	0	0	1	0	0	0
v_8	0	0	1	0	0	0	0	0