線性代數 試題 （限用答案本作答）

1. Consider a matrix $A=\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix}$
   (a) Find the eigenvalues and corresponding eigenvectors (15%)
   (b) Find a matrix P, such that $P^{-1}AP=D$, where D is a diagonal matrix, and write down D also. (5%)

2. Consider a subspace W with a basis $\{x_1, x_2, x_3\}$, where $x_1=\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, x_2=\begin{bmatrix}0\\1\\1\\1\end{bmatrix}, x_3=\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$.
   (a) Find an orthonormal basis for W. (15%)
   (b) Let $A=\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\\1&1&1\end{bmatrix}$, If A=QR, where the columns of Q are those obtained in (a), and R is an upper triangular matrix, write down R. (5%)

3. Let T be a linear transformation on R^2, and T(0,1)=(1,1), T(1,0)=(-1,1)
   (a) Find the image of $x^2+y^2=1$ (5%)
   (b) What's the area of this image? (5%)

4. Compute the following determinants.
   (a) $\begin{vmatrix}1-x&2&3&4\\1&2-x&3&4\\1&2&3-x&4\\1&2&3&4-x\end{vmatrix}=?$ (5%)
   (b) $\begin{vmatrix}a&b&b&b&b\\b&a&b&b&b\\b&b&a&b&b\\b&b&b&a&b\\b&b&b&b&a\end{vmatrix}=?$ (5%)

5. A linear system over R: $\begin{aligned}x-y+3z&=1\\-x+2y-3z&=4\\3x-3y+a^2z&=a\end{aligned}$, determine all values of a such that
   (a) The system has no solution (3%)
   (b) The system has infinitely many solutions (3%)
   (c) The system has an unique solution (4%)

6. Assume the matrix A is row equivalent to B. Find the following answers.
   (Col: column space, Row: row space, Nul: null space, dim: dimension)
   $A=\begin{bmatrix}1&-3&4&-1&9\\-2&6&-6&-1&-10\\-3&9&-6&-6&-3\\3&-9&4&9&0\end{bmatrix}, B=\begin{bmatrix}1&-3&0&5&-7\\0&0&2&-3&8\\0&0&0&0&5\\0&0&0&0&0\end{bmatrix}$
   (a) The basis for Col A. (5%)
   (b) The basis for Row A. (5%)
   (c) The basis for Nul A. (5%)
   (d) rank A = ? (3%)
   (e) dim Nul A = ? (2%)

7. Let T: $R^n \to R^m$ be a linear transformation. Prove that there exists an unique matrix A such that $T(x)=Ax$ for all x in R^n (10%)

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