

線性代數 試題

(限用答案本作答)

1. Consider a matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

- (a) Find the eigenvalues and corresponding eigenvectors (15%)  
 (b) Find a matrix P, such that  $P^{-1}AP = D$ , where D is a diagonal matrix, and write down D also. (5%)

2. Consider a subspace W with a basis  $\{x_1, x_2, x_3\}$ , where  $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

- (a) Find an orthonormal basis for W. (15%)

(b) Let  $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ , If  $A=QR$ , where the columns of Q are those obtained in (a), and R is an

upper triangular matrix, write down R. (5%)

3. Let T be a linear transformation on  $\mathbb{R}^2$ , and  $T(0,1)=(1,1)$ ,  $T(1,0)=(-1,1)$

- (a) Find the image of  $x^2 + y^2 = 1$  (5%)  
 (b) What's the area of this image? (5%)

4. Compute the following determinants.

(a)  $\begin{vmatrix} 1-x & 2 & 3 & 4 \\ 1 & 2-x & 3 & 4 \\ 1 & 2 & 3-x & 4 \\ 1 & 2 & 3 & 4-x \end{vmatrix} = ?$  (5%)

(b)  $\begin{vmatrix} a & b & b & b & b \\ b & a & b & b & b \\ b & b & a & b & b \\ b & b & b & a & b \\ b & b & b & b & a \end{vmatrix} = ?$  (5%)

$x - y + 3z = 1$

5. A linear system over  $\mathbb{R}$ :  $-x + 2y - 3z = 4$ , determine all values of a such that

$3x - 3y + a^2z = a$

- (a) The system has no solution (3%)  
 (b) The system has infinitely many solutions (3%)  
 (c) The system has an unique solution (4%)

6. Assume the matrix A is row equivalent to B. Find the following answers.

(Col: column space, Row: row space, Nul: null space, dim: dimension)

- (a) The basis for Col A. (5%)

- (b) The basis for Row A. (5%)

- (c) The basis for Nul A. (5%)

- (d) rank A = ? (3%)

- (e) dim Nul A = ? (2%)

$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

7. Let  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation. Prove that there exists a unique matrix A such that  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^n$  (10%)

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