

微積分試題 (限用答案本作答)

所有答案請填寫於答案本內，禁止使用電子計算機

壹、 Find the following limits: (10 分)

(a)  $\lim_{x \rightarrow 0^+} (x+1)^{\cot x} = \underline{\hspace{2cm}}$       (b)  $\lim_{x \rightarrow 0} \frac{\ln(3+x) - \ln(3)}{x} = \underline{\hspace{2cm}}$

貳、 Evaluate the following integrals: (15 分)

(a)  $\int_{\pi^2/9}^{\pi^2/4} \frac{\cos \sqrt{x}}{\sqrt{x}} dx = \underline{\hspace{2cm}}$       (b)  $\int_{-\pi}^{\pi} (\sin^3 x + x \cos^4 x) dx = \underline{\hspace{2cm}}$

(c)  $\int_0^1 \int_0^2 2ye^x dx dy = \underline{\hspace{2cm}}$

參、 Suppose that  $f(2)=1, f'(2)=2, f''(2)=3, g(2)=2, g'(2)=-1$ . Find each value

(a)  $D_x^2(f^2(x))$  at  $x=2$       (b)  $\frac{d}{dx}(f(x) \cdot g(x))$  at  $x=2$

(c)  $\frac{d}{dx}(f(g(x)))$  at  $x=2$  (15 分)

肆、 Sketch the graph of function  $f$ . (10 分)

$$f(x) = \frac{x^3 - 2x^2 - 3x}{x^2 - 1}$$

伍、 If  $\frac{dy}{dx} = \sin x + xe^x + e^x$ , and  $y = 2$  at  $x = 0$ . Find the function  $y$ . (10 分)

陸、 Find the volume of the solid generated by revolving the region bounded by the parabolas  $y = x^2$  and  $y = 3x$  about the  $y$ -axis. (10 分)

柒、 Indicate whether  $\sum_{k=1}^{\infty} \frac{1}{k^2 + 2k}$  converges or diverges. If it converges, find its sum. (10 分)

捌、 Prove or disprove that (10 分)

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

玖、 Represent  $\sqrt{1+x^3}$  in a Maclaurin Series and use it to approximate

$\int_0^1 \sqrt{1+x^3} dx$  to six decimal place. (10 分)

(Hint:  $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \dots$ )

試題完