銘傳大學 95 學年度應用統計資訊學系碩士班招生考試

第一節　　　（第 1 頁共 1 頁）

機率論試題　　　（限用答案本作答）

1. (20 points) Given the cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^2 + 0.2 & \text{for } 0 \le x < 0.5 \\ x & \text{for } 0.5 \le x < 1 \\ 1 & \text{for } 1 \le x \end{cases}$$

(a) Find $P\,[0.2 < X \le 0.6\,]$.

(b) Find the probability density function of X.

2. (20 points) Suppose X has a discrete density function given by

$$f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots$$

(a) Find $\mathrm{E}\,(t^X)$.

(b) Please use the result in (a) to find $\mathrm{E}\left[X(X-1)(X-2)\right]$.

3. (20 points) Let $X_1, \ldots, X_n$ be a random sample from the probability density function

$$f\,(x;\theta) = \theta\, e^{-\theta x} \quad I_{[0,\infty)}(x)$$

(a) Find a function of $\sum_{i=1}^{n} X_i$, $g\,(\sum_{i=1}^{n} X_i)$, such that $\mathrm{E}\,[g\,(\sum_{i=1}^{n} X_i)] = \theta$.

(b) Define $Y_1 = \min\,[\,X_1, \ldots, X_n\,]$. Find an unbiased estimator of $\frac{1}{\theta}$ based only on Y_1.

4. (10 points) Let $X_1, \ldots, X_n$ be a random sample from $N\,(\theta, 1)$. What is the maximum-likelihood estimator of P [X>1].

5. (30 points) Let X_1, X_2 be a random sample of size 2 from $N\,(0, 1)$ and Y_1, Y_2 a random sample of size 2 from $N\,(2, 1)$. Suppose the X_1, X_2 are independent of the Y_1, Y_2. What are the distributions of the following random variables?

(a) $\overline{X} + \overline{Y}$.

(b) $(X_1 + X_2)^2 / (X_1 - X_2)^2$.

(c) $(X_1 + X_2) / \sqrt{(X_2 - X_1)^2}$.

(d) $[(Y_1 - Y_2)^2 + (X_1 - X_2)^2 + (X_1 + X_2)^2]/2$.

(e) $(Y_1 + Y_2 - 4)^2 / (Y_1 - Y_2)^2$.

試題完