銘傳大學 97 學年度研究所碩士班招生考試

應用統計資訊學系碩士班

機率論試題(第三節)

(第 1 頁共 1 頁)(限用答案本作答)　　□可使用計算機 ☑不可使用計算機

1. (20 points) Suppose that random variable $X$ is uniformly distributed distribution over the interval (0, 1); that is, $f_x(x) = I_{(0,1)}(x)$. Assume that the conditional distribution of $Y$ given $X = x$ has a binomial distribution with parameters $n$ and $p = x$; i.e.,

$P[Y = y \mid X = x] = \binom{n}{y} x^y (1-x)^{n-y}$ for $y = 0, 1, \ldots, n$.

(a) Find $E[Y]$.

(b) Find the distribution of $Y$.

2. (12 points) Let $f_{X,Y}(x,y) = e^{-(x+y)} I_{(0,\infty)}(x) I_{(0,\infty)}(y)$. Find $P[2<X+Y<3]$ and $P[X<Y|X<3Y]$.

3. (12 points) Let $X$ be a random variable with probability density function given by

$$f_x(x) = |1-x| I_{[0,2]}(x).$$

Find the mean and variance of $X$.

4. (12 points) Let $X_1$, $X_2$ and $X_3$ be uncorrelated random variables with common variance $\sigma^2$. Find the correlation coefficient between $X_1 + X_2$ and $X_2 + X_3$.

5. (12 points) If $X$ has a normal distribution with mean $\mu$ and variance $\sigma^2$, find the distribution, mean, and variance of $Y = e^x$.

6. (12 points) Derive the mean of the $F_{m,n}$ distribution.

7. (20 points) Let $X_1, X_2, \ldots, X_n$ be a random sample from the probability density function $f(x) = \frac{1}{\beta} e^{-x/\beta}, 0 < x < \infty$.

(a) Find a function of $X_1, X_2, \ldots, X_n$, $g(X_1, X_2, \ldots, X_n)$, such that $E[E[g(X_1, X_2, \ldots, X_n)]] = 1/\beta$.

(b) Find **two** unbiased estimators of $\beta^2$.

試題完