

銘傳大學八十九學年度轉學生招生考試

八月一日 第五節

資管 轉三

管理數學 試題

請按照題號依序橫式作答，並請將答案標示清楚。

一、Let  $A$  be the following matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 2 & 0 \\ 3 & 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- (1) Find a basis for the row space of  $A$ . 5%
- (2) Find a basis for the column space of  $A$ . 3%
- (3) Find a basis for the null space of  $A$ . 5%
- (4) Verify the dimension theory of the matrix  $A$ . 2%

二、Solving the following linear system

$$3x + 2y - z = -1$$

$$x - y - z = 0$$

$$2x + y - 2z = 3$$

- (1) By Gauss-Jordan elimination. 10%
- (2) By Cramer's rule. 10%

三、Let  $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$

- (1) Find the eigenvalue and eigenvector of  $A$ . 10%
- (2) Use part (1) to compute  $A^{10}$ . 5%

- 四、(1) Use the Gram-Schmidt process to transform the basis  $\{(1,1,1), (0,1,1), (1,2,3)\}$  for  $\mathbb{R}^3$  into an orthonormal basis for  $\mathbb{R}^3$ . 10%
- (2) Write  $(2,3,1)$  as a linear combination of the basis obtained in part (1). 5%

五、Let  $W$  be all  $3 \times 3$  symmetric matrices

- (1) Show that  $W$  is a subspace of  $M_{33}$ .      5%  
(2) Find a basis for  $W$ .      10%

六、 Let  $L : R^n \rightarrow R^m$  be a linear transformation defined by  $L(x) = Ax, x \in R^n$   
where  $A$  is an  $m \times n$  matrix

- (1) Show that  $L$  is one-to-one if and only if  $\text{rank}A = n$ .      10%  
(2) Show that  $L$  is onto if and only if  $\text{rank}A = m$ .      10%

試題完