

銘傳大學九十一學年度轉學生招生考試

八月四日 第四節

應統 轉三

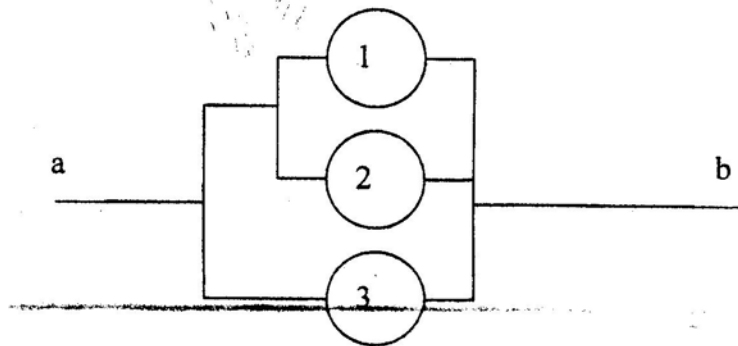
應用機率論 試題

* 可使用計算機

1. (10%) Consider the following segment of an electric circuit(電路) with three relays. Current(電流) will flow from a to b if there is at least one closed path when the relays are switched to "closed." However, the relays may malfunction(故障). Suppose they close properly only with probability 0.9 when the switch is thrown, and suppose they operate independently of one another.

(a) Find the probability of the event that current will flow from a to b when the relays are switched to "closed."

(b) Find the probability that relay 1 closed properly, given that current is known to be flowing from a to b.



2. (10%) Let X be a random variable whose distribution function F is given by

$$F(x) = P(X \leq x) = \begin{cases} 0, & x < 0 \\ x/3, & 0 \leq x < 1 \\ x/2, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases} \quad \text{Find (a) } P\left(\frac{1}{2} \leq X \leq 1\right) \quad \text{(b) } P\left(\frac{1}{2} \leq X < 1\right)$$

3. (10%) A sugar refinery (精練場) has three processing plants(工廠), all receiving raw sugar(粗糖) in bulk. The amount of sugar that one plant can process in one day can be modeled as having an exponential distribution with a mean of 4

(measurements in tons) for each of the three plants.

- (a) Find the probability that any given plant processes more than 4 tons.
- (b) If the plants operate independently, find the probability that exactly two of the three plants process more than 4 tons on a given day.

4. (10%) If $f(x_1, x_2) = \left(\frac{2}{3}\right)^{x_1+x_2} \left(\frac{1}{3}\right)^{2-x_1-x_2}$, $(x_1, x_2) = (0,0), (0,1), (1,0), (1,1)$, zero

elsewhere, is the joint probability function of X_1 and X_2 .

- (a) Find the conditional probability function of X_1 given $X_2=1$.
 - (b) Find the joint probability function of $Y_1=X_1-X_2$ and $Y_2=X_1+X_2$.
5. (20%) With X_1 denoting the amount of gasoline(汽油) stocked in a bulk tank(貯存槽) at the beginning of a week and X_1 the amount sold during the week, $Y=X_1-X_2$ represents the amount left over at the end of the week.(即 X_1 是每週剛開始的油量, X_2 是賣掉的, Y 是剩下的) If the density function of (X_1, X_2) is given by $f(x_1, x_2) = \begin{cases} 3x_1 & 0 \leq x_2 \leq x_1 \leq 1 \\ 0 & \text{otherwise} \end{cases}$
- (a) Find the covariance of X_1 and X_2 .
 - (b) Find the probability density function of Y .

6. (10%) A certain retailer(零售商) for a petroleum product sells a random amount, X , each day. Suppose that X , measured in hundreds of gallons(單位為 100 加侖), has the probability density function

$$f(x) = \begin{cases} 3x^2/8 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

The retailer's profit(收益) turns out to be \$5 for each 100 gallons sold if $X < 1$ and \$8 per 100 gallon if $X > 1$. Find the retailer's expected profit for any given day.

7. (10%) Let X and Y be independent random variables each having the uniform distribution over the interval $(0,1)$. Find the conditional density function of X given $Z=z$, where $Z=X+Y$.
8. (20%) Let X and Y be two independent random variables having the respective normal densities $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. Then
- (a) Find the moment generating function of X . (hint: $E(e^{tX})$).
 - (b) Prove that $X+Y$ has the normal density $N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

試題完